Numerical solutions for transient natural convection in a square cavity with different sidewall temperatures

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A comprehensive numerical study was made of the transient natural convection in a square cavity at large Rayleigh numbers. The flow was initiated by instantaneously raising and lowering the temperatures on the opposing sidewalls. Extensive and systematic numerical solutions to the two-dimensional, time-dependent Navier-Stokes equations were carried out. The main thrust was to examine the effect of the Prandtl number and of the thermal conditions on the horizontal surfaces. Attention was focused on the time histories of temperature and velocity. The time dependence of the Nusselt number was also determined. When $Pr \geq 1$, a distinct oscillatory behavior is oberved if the criterion $Ra > Pr^4 A^{-4}$ is strongly satisfied, where A $[=1.0]$ is the cavity aspect ratio. The computed period of oscillation is compatible with the period for internal gravity waves, lending support to the predictions of Patterson and Imberger. The influence of the thermal boundary conditions on the horizontal surfaces appears to have a negligible influence on the time histories.

Keywords: transient natural convection; internal gravity wave; horizontal surface conditions

Introduction

The flow and heat transfer characteristics of buoyancy-driven fluid motions inside a closed cavity have received considerable attention (see, e.g., the review article by Ostrach[1]). In addition to such well-known areas of application in thermal engineering, the subject matter is increasingly relevant to modern technological innovations (e.g., crystal growth and materials processing in space, etc.). As is the case for the majority of these applications, attention will be given primarily to situations in which the system Rayleigh number Ra is sufficiently large so that convection is predominant. Also, attention is limited to enclosures with an aspect ratio of the order of unity.

In an effort to isolate and clearly identify the main physics involved, the flow inside a rectangular cavity with differentially heated vertical sidewalls has previously been investigated extensively. These studies have mostly dealt with the steady-state regime. Despite the practical importance of time-dependent convective situations in nature as well as in engineering, published accounts of unsteady cavity convection have been relatively scarce.[2–7]

The recent paper, Ref. 2, improved our understanding of transient behavior of natural convection in a two-dimensional rectangular cavity. Their analysis centered on the flow of an initially isothermal fluid ($\theta=\theta_0$) driven by abruptly raising ($\theta_h=\theta_0+\Delta\theta/2$) and lowering ($\theta_c=\theta_0-\Delta\theta/2$) the respective vertical sidewall temperatures of the rectangle. Relying heavily on scaling arguments and physical insights, Patterson and Imberger provided broad classifications of the transient regimes of flow in terms of several nondimensional parameters. A key contention which emerged from their analysis was the existence of a decaying oscillatory approach to steady state; they stated that this behavior reflected the result of transient system-scale internal wave activity. It was shown that the criterion for such oscillatory behavior was $Ra > Pr^4 A^{-4}$, where Pr is the Prandtl number and A the aspect ratio (height/width). Patterson and Imberger presented a few illustrative numerical solutions for a square cavity ($A=1$), and their results were shown to be compatible with their theoretical expositions.

The presence of the internal waves in transient cavity convection has been a topic of intense discussion in the recent literature. Prior work has been reported on the closely related problem of heat-up (or heat-down); this refers to the transient process of fluid adjustment from a given state of stratification to a new state of stratification due to the alteration in thermal boundary condition on the walls of the entire enclosure. For this type of process in a cylinder, the overall approach to the steady state is accomplished over a time scale $Ra^{1/4} N_d^{-1}$, where N_d is the dimensional Brunt-Vaisala frequency of the new state of stratification.[8–12] Numerical solutions to the full Navier-Stokes equations for a cylindrical container of aspect ratio of order unity revealed that the velocity field evolved in a highly oscillatory fashion, the frequency being scaled with N_d.[10–13] The laboratory experiment on cool down in a cylinder of aspect ratio of order unity by Otis and Roesler[14] also displayed wave motions, and the observed frequency was closed to the buoyant frequency N_d of the system.

The issue of the internal gravity waves does not appear to have been completely resolved for the pattern of flow envisioned by Ref. 2. Laboratory measurements employing shallow cavities ($A=0.0625$, 0.0112) showed no evidence of waves, pointing to an apparent discrepancy between theory and experiment.[6] Later, Patterson[15] proposed more detailed orderings for the classification of the flow, and asserted that the experiments by Ref. 6 were actually performed in a regime in which internal wave activity would not be expected. Ivey[3] conducted similar experiments in a square cavity ($A=1$) at Rayleigh numbers ($\sim 10^9$), which were much higher than the values ($\sim 10^5$) used in the numerical computations of Ref. 2. Ivey's experiments detected oscillation in temperature in the corners of the cavity,

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Received 1 December 1987; accepted for publication 19 September 1988

but Ivey attributed this phenomenon to an internal hydraulic jump.

Stimulated by the aforementioned recent interest in transient convection in a cavity, one powerful approach appears to be more comprehensive and explicit numerical solutions. The exemplary numerical results of Ref. 2 corroborated the qualitative global features obtainable from their scale analyses; however, further systematic and extensive numerical runs are needed to verify the details of the theory. Such specific and illustrative numerical results might clarify the nature of the transient oscillatory behavior predicted by Ref. 2.

Numerical solutions have been secured for the time-dependent two-dimensional Navier-Stokes equations for $A=1$. A broad range of the parametric values was encompassed such that the criterion proposed by Ref. 2 for the wave activity could be tested. Since the aspect ratio was set to be unity, the principal thrust of the study was to ascertain the effect of the Prandtl number on the transient process. Numerical solutions using extreme values of Pr and $\mathrm{Pr} \sim O(1)$ have been carried out. The time evolving structures of flow and thermal fields were scrutinized, and the heat transfer rate on the boundary was computed. The calculations covered a range of $\mathrm{Pr}=0.025$–100, $\mathrm{Ra}=10^4$–10^7.

Another motivation for this paper is to investigate the effect of the thermal conditions at the horizontal boundaries. For the geometries considered by Refs. 2, 3, and 6, the upper and lower horizontal walls were thermally insulated. Prior work on the natural convection in a cavity with destabilizing, thermally conducting horizontal surfaces has not been numerous. Of more practical importance is a linear variation in temperature on the horizontal walls. These conditions on the horizontal wall were considered in the transient analysis by Hyun and Lee[7] for a fluid with temperature-dependent viscosity. Briggs and Jones[16] studied steady-state natural convection experimentally at high Rayleigh numbers in a square cavity equipped with such horizontal walls. The numerical solutions to the two-dimensional transient Navier-Stokes equations incorporating the linearly varying horizontal temperature conditions were obtained in the present paper. This problem was first addressed in the classical paper by Ref. 5; because of the limited computing power at that time, the parameter space considered in that paper was rather restricted. The aim here was to calculate the time-dependent flow in an extended parameter space, and compare the results with those obtained for insulating horizontal surfaces. Descriptions are presented of the numerically constructed evolution of flow in the cavity. These numerical results serve as background flow data to illuminate the specific influence of the thermal condition at the horizontal surfaces on the interior fluid motion.

In summary, as a sequel to Ref. 7, this paper has the objective of depicting the highlights of time histories based on the numerical solutions. The information presented in the present paper may be used as standards against which future laboratory measurements can be compared.

The model

The transient two-dimensional fluid motions under present consideration are governed by the Navier-Stokes equations with the Boussinesq assumption. In the usual vorticity (ζ)–stream function (ψ) formulation, and expressed in dimensionless form, these are

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$\frac{\partial T}{\partial \tau}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2} \tag{2}$$

$$\frac{\partial \zeta}{\partial \tau}+U\frac{\partial \zeta}{\partial X}+V\frac{\partial \zeta}{\partial Y}=\mathrm{Pr}\left[\frac{\partial^2 \zeta}{\partial X^2}+\frac{\partial^2 \zeta}{\partial Y^2}\right]+\mathrm{Ra}\,\mathrm{Pr}\frac{\partial T}{\partial X} \tag{3}$$

$$\frac{\partial^2 \psi}{\partial X^2}+\frac{\partial^2 \psi}{\partial Y^2}=-\zeta \tag{4}$$

$$U=\frac{\partial \psi}{\partial Y}, \qquad V=-\frac{\partial \psi}{\partial X} \tag{5}$$

In the above, the nondimensional quantities are defined as

$$X=\frac{x}{h}, \quad Y=\frac{y}{h}, \quad U=\frac{u}{\kappa/h}, \quad V=\frac{v}{\kappa/h}$$

$$T=\frac{\theta-\theta_0}{\theta_h-\theta_c}, \quad \tau=\frac{t}{h^2/\kappa}, \quad \mathrm{Pr}=\frac{\nu}{\kappa}$$

$$\mathrm{Ra}=\frac{g\beta\,\Delta\theta h^3}{\kappa\nu}$$

Figure 1 is the schematic of the configuration. The square cavity is of width and height h, and the Cartesian coordinates

Notation

A	Aspect ratio, height/width
C_p	Specfic heat
h	Height of cavity
k	Thermal conductivity
N	Nondimensional Brunt-Väisälä frequency
N_d	Dimensional Brunt-Väisälä frequency
Nu	Average Nusselt number over boundary
Pr	Prandtl number, ν/κ
Ra	Rayleigh number, $g\beta\,\Delta\theta h^3/\kappa\nu$
t	Time
T	Nondimensional temperature
u, v	Velocity components
U, V	Nondimensional components of velocity
x, y	Coordinates
X, Y	Nondimensional coordinates
β	Coefficient of volumetric expansion with temperature
ζ	Nondimensional vorticity
θ	Dimensional temperature
κ	Thermal diffusivity, $k/\rho C_p$
ν	Kinematic viscosity
ρ	Density
τ	Nondimensional time
τ_h	Nondimensional time-scale for heatup, $\mathrm{Ra}^{-1/4}$
τ_g	Nondimensional period of oscillation for internal gravity waves, obtainable from $2\pi(1+A^2)^{1/2}/N$
τ_0	Nondimensional period of oscillation observed in the numerical solutions
ψ	Nondimensional stream function

Subscripts

0	Reference value
c	Cold wall
h	Hot wall

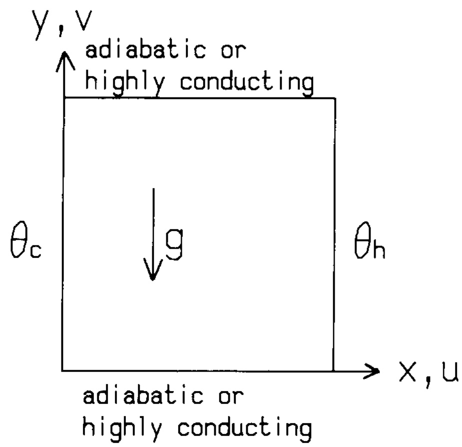

Figure 1 Configuration and coordinate system

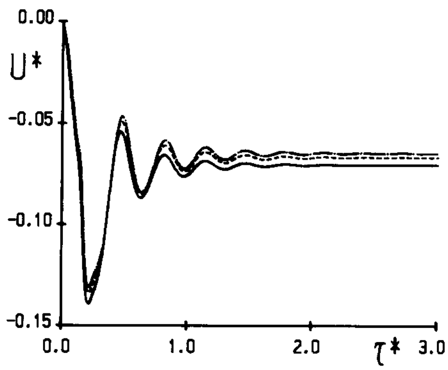

Figure 2 Plots of horizontal velocity U^* $(=u\,\mathrm{Ra}^{-1/2}h/\kappa)$ versus scaled time τ^* $(=\mathrm{Ra}^{1/4}\tau)$ at $X=0.5$, $Y=0.8$: ——— (31 × 31) mesh points; ------ (41 × 41) mesh points; —·—·— (51 × 51) mesh points. Conditions are $\mathrm{Ra}=10^6$, $\mathrm{Pr}=1$. Adiabatic horizontal wall conditions

(x, y) with the corresponding velocity components (u, v) are indicated therein. The relevant fluid properties are ν, the kinematic viscosity, κ, the thermal diffusivity, and β, the coefficient of volumetric expansion with temperature. In line with the customary practice, the fluid properties are taken to be constant, and dissipation is neglected in the energy equation. The principal nondimensional parameters are the Prandtl number Pr and the Rayleigh number Ra.

The appropriate boundary conditions are

$u=v=0$ on all solid boundaries
$T=-0.5$ on $X=0$, $T=0.5$ on $X=1$
$\partial T/\partial Y=0$ on $Y=0, 1$ (adiabatic horizontal walls)
$T=X-0.5$ on $Y=0, 1$ (conducting horizontal walls)

Numerical techniques to solve the above equations have been well established. We have chosen an amended version of finite-difference methods developed by Ref. 4. Sensitivity tests to grid size for this particular algorithm were previously carried out in detail by Ref. 4. In order to check the possibility that the oscillatory behavior may be significantly influenced by the choice of grid size, similar tests for the present numerical solutions were performed. A representative result of such tests is displayed in Figure 2. The velocity traces indicate that numerical solutions are insensitive to the grid size. Based on these tests, the mesh points employed in this paper were typically (31 × 31). The reader is referred to Ref. 4 for the specifics of the numerical model.

Results and discussion

The first series of computations is for the case of insulated horizontal surfaces. Previous studies have clearly established the general pattern of flow; therefore, the impetus of the present investigation will be directed toward the time histories of the interior flow and temperature fields as well as the rate of heat transfer on the cavity walls. Time records are essential to estimate the time scale for global adjustment and to detect the existence of an oscillatory approach to the steady state.

Guided by the previous analyses of Ref. 2 and others, the transient results for the flow regimes $\mathrm{Pr}\geq 1$ are first described.

Figure 3 illustrates the evolution of temperature and velocity at several selected points for $\mathrm{Ra}=10^6$, $\mathrm{Pr}=100$. Note that this case belongs to the regime $\mathrm{Ra}<\mathrm{Pr}^4A^{-4}$, as classified by Ref. 2. When the horizontal surfaces are insulated, the overall flow structure is antisymmetric about the diagonal line of the cavity; therefore, Figure 3 presents the transient records at the points located in the upper region and in the right region (close to the hot wall). It is immediately evident in the temperature traces of Figure 3 that no oscillatory behavior occurs. This qualitative feature is consistent with the predictions of Ref. 2. Inspection of the results in Figure 3 clearly suggests that the global process of adjustment is substantially accomplished over a time scale τ_h, which scales with $O(\mathrm{Ra}^{-1/4})$. The observation that $\tau_h \sim O(\mathrm{Ra}^{-1/4})$ is in agreement with the theoretical findings for the heat-up process (see Refs. 8–12). The velocity traces are

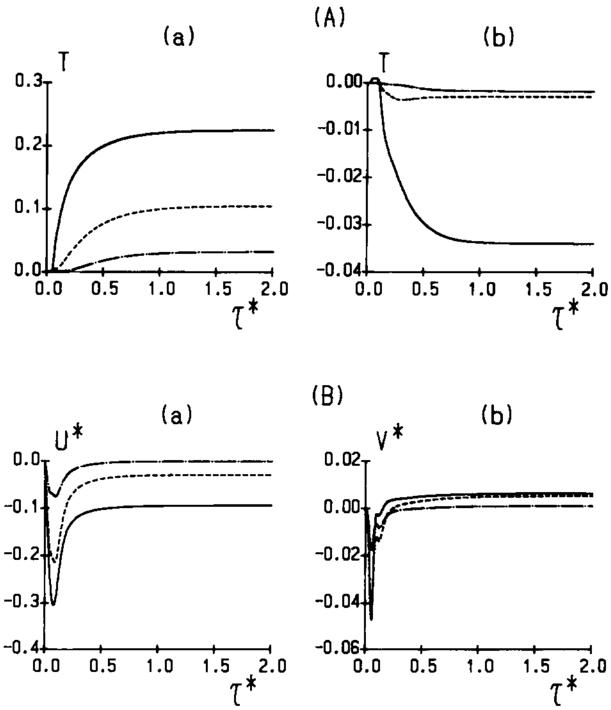

Figure 3 (A) Plots of nondimensional temperature T versus scaled time τ^* at (a) $X=0.5$: ——— $Y=0.833$; ------ $Y=0.700$; —·—·— $Y=0.567$. (b) $Y=0.5$: ——— $X=0.833$; ------ $X=0.700$; —·—·— $X=0.567$; (B) plots of horizontal velocity U^* versus scaled time τ^* (shown in (a)) and vertical velocity V^* versus scaled time τ^* (shown in (b)). The horizontal position is at $X=0.5$: ——— $Y=0.833$; ------ $Y=0.700$; —·—·— $Y=0.567$. Conditions are $\mathrm{Ra}=10^6$, $\mathrm{Pr}=100$. Adiabatic horizontal wall conditions

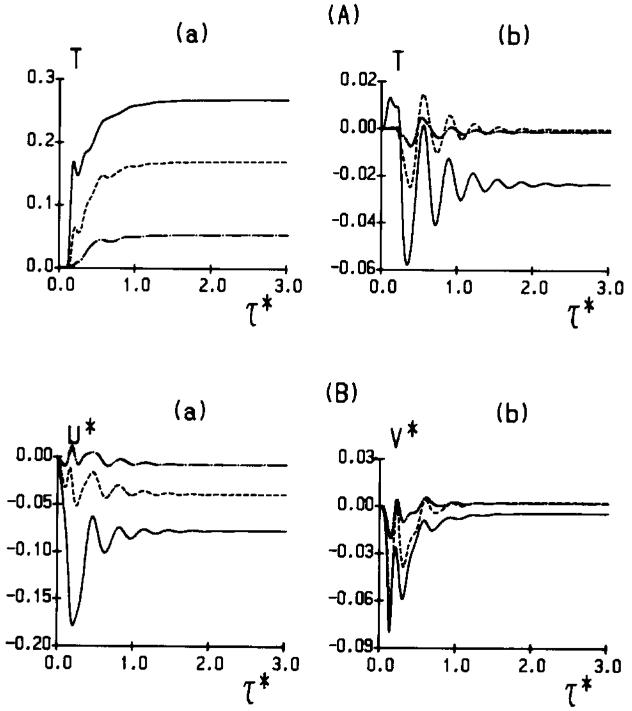

Figure 4 Same as in Figure 3. Conditions are Ra = 10^6, Pr = 1. Adiabatic horizontal wall conditions

equally smooth, and no oscillatory behavior is noticed at this value of Pr, which is in accord with the results of Hyun[17] for heat-up. The characteristics of flow and temperature shown in Figure 3 exemplify the regime $Ra < Pr^4 A^{-4}$.

Figure 4 is for $Ra = 10^6$, $Pr = 1$, which belongs to the regime $Ra > Pr^4 A^{-4}$. The transient flow is highly oscillatory at small and intermediate times. Only near the steady-state limit, the oscillations are damped out. A closer inspection of the time records indicates that the period of oscillation τ_0 is 0.010, and the oscillation survives for several periods. According to Ref. 2, the period of the systemwide internal gravity waves was scaled with τ_g; i.e.,

$$\tau_g = 2\pi(1 + A^2)^{1/2}/N \qquad (6)$$

where $N = (Ra\,Pr)^{1/2}$ is the Brunt-Väisälä frequency nondimensionalized by use of κ/h^2. For the run of Figure 4, $\tau_g = 0.0089$; the close agreement between the observed τ_0 and the predicted τ_g lends support to the theory of Ref. 2. As mentioned before, the global adjustment is attained over the time scale $\tau_h \sim O(Ra^{-1/4})$. Several more runs were made for $Ra = 10^6$ and using values of Pr larger than 1 but still satisfying $Ra > Pr^4 A^{-4}$. For instance, for $Ra = 10^6$ and $Pr = 2$, a distinct transient oscillatory activity, with period $\tau_0 = 0.0076$ was seen. The value of τ_g from Equation 6 for this case was 0.0063.

Figure 5 shows two runs for $Ra = 10^5$ and for two different Prandtl numbers; i.e, (A) Pr = 1 and (B) Pr = 10. The criterion $Ra > Pr^4 A^{-4}$ is still satisfied. Oscillatory behavior is less pronounced in Figure 5(A) than in Figure 4. Note that the criterion $Ra > Pr^4 A^{-4}$ is less strongly satisfied for the case shown in Figure 5(A) than that in Figure 4. The oscillatory period τ_0 assessed from Figure 5(A) is 0.030–0.034, while τ_g from (Equation 6) is 0.028. Figure 5(B) demonstrates the results of a marginal case (i.e., $Ra\,Pr^{-4} A^{-4} = 10$), which indicates that the product of the parameters does not greatly exceed the critical value of unity. The traces of temperature and horizontal velocity exhibit no evidence of oscillatory motion. This run was designed to validate the experimental measurements of Ref. 3 for which $Ra\,Pr^{-4} A^4 = 8.6$. The present results shown in Figure 5(B) are in qualitative consistency with the measurements of Ref. 3.

The data displayed in Figures 4 and 5 point to the observation that, when the value of Ra exceeds well above $Pr^4 A^{-4}$, a distinct transient oscillatory behavior is discernible. The period of these oscillations is comparable to the period of internal gravity waves given by Equation 6. These findings based on the numerical results are clearly corroborative of the theoretical predictions by Ref. 2.

Attention is now turned to $Pr < 1$. The majority of the prior work was undertaken for $Pr \geq 1$, but Hyun[17] examined the effect of $Pr < 1$ in the context of heat-up process in a cylinder.

Figure 6 illustrates two runs for $Pr < 1$. Note that the criterion $Ra > Pr^4 A^{-4}$ is trivially satisfied for $Pr < 1$. Distinct transient oscillations are apparent in Figure 6(A) for $Ra = 10^6$ and $Pr = 0.2$. The period of oscillation detectable in Figure 6(A) is $\tau_0 = 0.018$–0.022, while τ_g from Equation 6 is 0.020. Figure 6(B) shows the result for $Ra = 10^4$, $Pr = 0.1$. The criterion $Ra > Pr^4 A^{-4}$ is still satisfied; however the value of Ra is relatively small so that only weak internal wave motions would be expected. The flow evolves mostly in a monotonic manner.

The second phase of computations was conducted by adopting conducting horizontal surfaces having a linearly varying temperature profile. Figure 7 displays the time record for two runs; i.e., one run (shown in (A)) for $Ra = 10^6$, $Pr = 1$, and the other run (shown in (B)), for $Ra = 10^5$, $Pr = 10$. As far as the time histories are concerned, the overall patterns of evolution for the conducting horizontal walls, as captured in Figure 7, are qualitatively similar to those for the adiabatic horizontal walls, as previously shown in Figures 4 and 5. One significant difference is the period of oscillation. Inspection of large amounts of numerical data leads to the conclusion that the period of oscillation for the conducting walls is greater than that for the insulating walls. For instance, for $Ra = 10^6$, $Pr = 1$ and 2, the periods of oscillation are 0.012 and 0.0088, respectively.

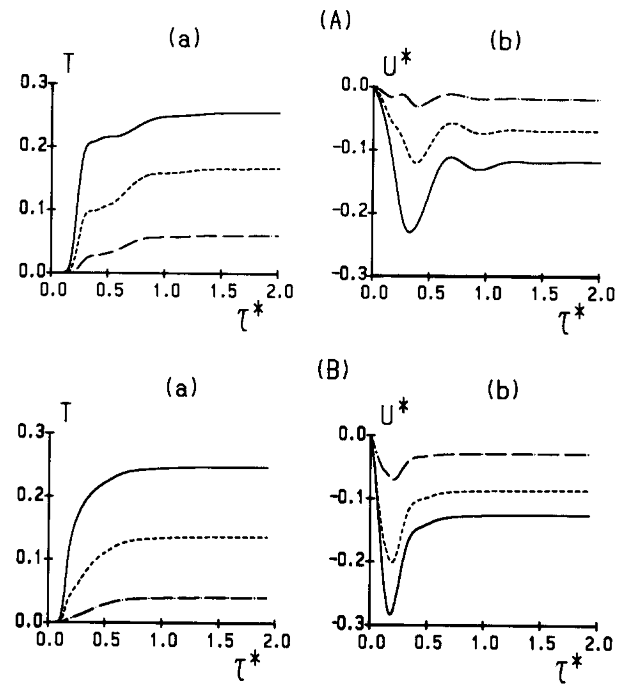

Figure 5 Plots of nondimensional temperature *T* versus scaled time τ^* (shown in (a)) and horizontal velocity U^* versus scaled time τ^* (shown in (b)). The horizontal position is at $X = 0.5$: —— $Y = 0.833$; ------ $Y = 0.700$; —·—·— $Y = 0.567$. Conditions are (A) Ra = 10^5, Pr = 1, (B) Ra = 10^5, Pr = 10. Adiabatic horizontal wall conditions

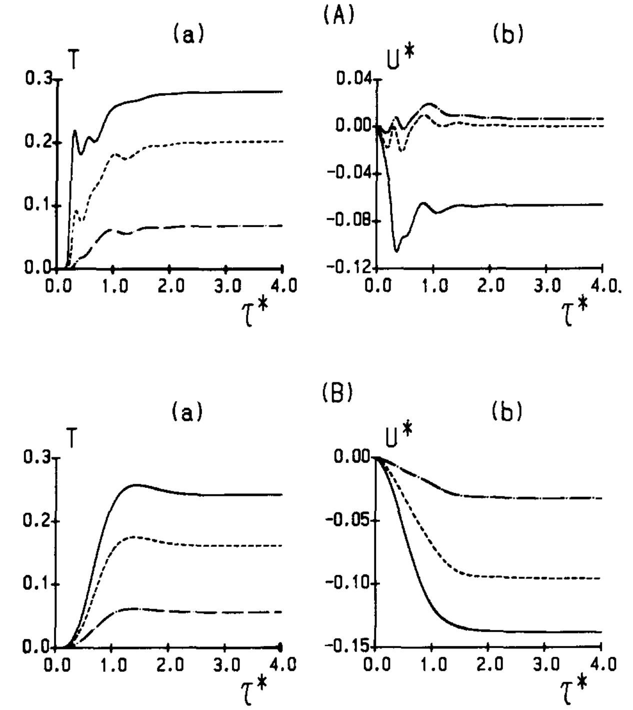

Figure 6 Same as in Figure 5. Conditions are (A) Ra = 10⁶, Pr = 0.2, (B) Ra = 10⁴, Pr = 0.1. Adiabatic horizontal wall conditions

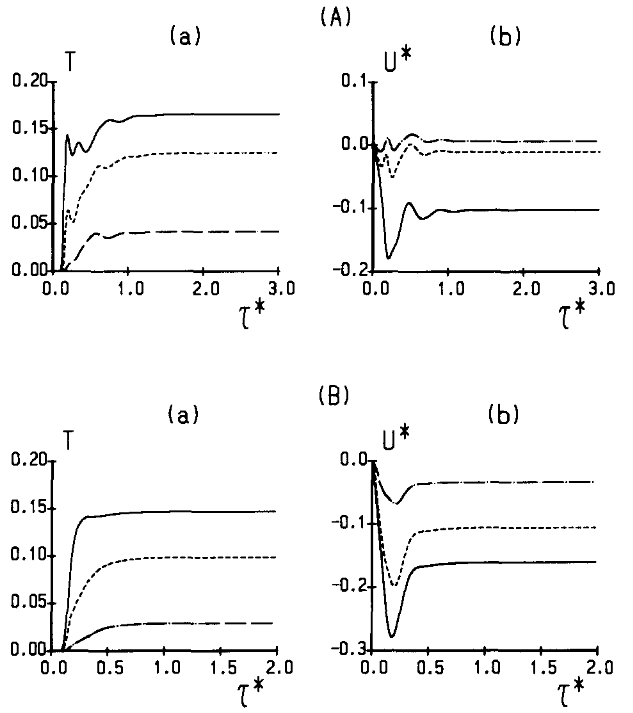

Figure 7 Same as in Figure 5. Conditions are (A) Ra = 10⁶, Pr = 1, (B) Ra = 10⁶, Pr = 10. Conducting horizontal wall conditions

These values are greater than 0.010 and 0.0076, as previously stated, for the insulating walls. The conducting walls permit heat exchange between the fluid and the bounding surfaces; therefore, the fluid stratification is, in general, weaker for conducting walls than for adiabatic walls. Also, for comparison purposes, the time histories records at a higher Rayleigh number, $Ra = 10^7$ and $Pr = 1$, for the two horizontal wall conditions are presented in Figure 8. Highly oscillatory temperature and flow fields are noticeable. The periods of oscillation are $\tau_0 = 0.0039$ for conducting horizontal walls and $\tau_0 = 0.0034$ for insulating horizontal walls, while τ_g from Equation 6 is 0.0028.

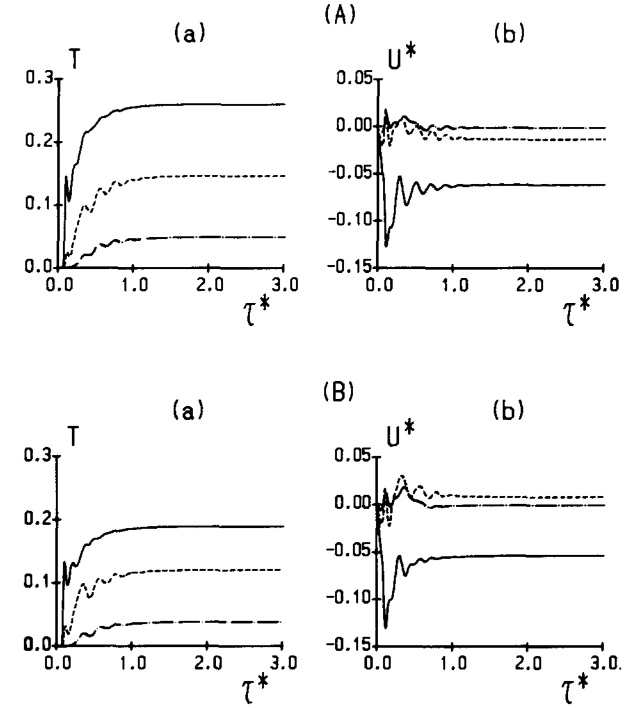

Figure 8 Same as in Figure 5. Conditions are Ra = 10⁷, Pr = 1: (A) Adiabatic horizontal wall conditions and (B) conducting horizontal wall conditions

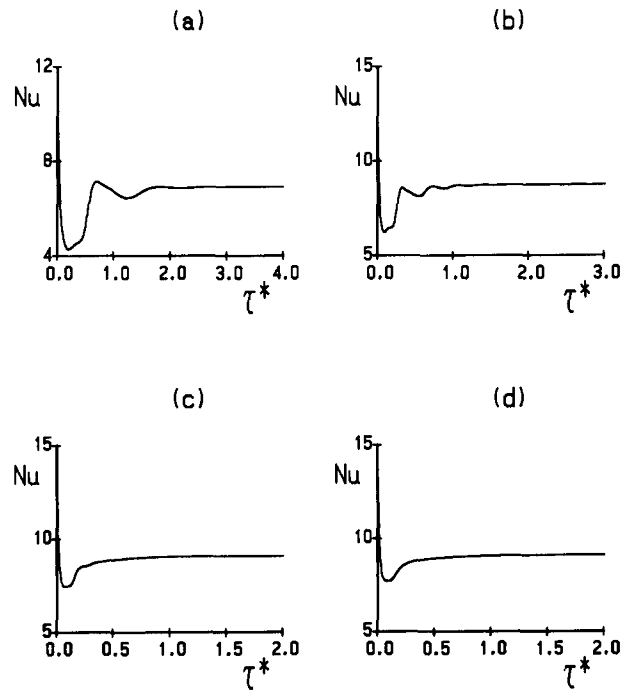

Figure 9 Plots of the mean Nusselt number Nu versus scaled time τ^* for $Ra = 10^6$ and (a) $Pr = 0.1$, (b) $Pr = 1$, (c) $Pr = 10$, (d) $Pr = 100$. Adiabatic horizontal wall conditions

Table 1 Average steady-state Nusselt numbers based on the numerical solutions

Ra	Pr	Nu	Ra	Pr	Nu
10^4	0.1	2.048	10^5	100.	4.929
10^4	1.	2.316	2×10^5	0.2	5.035
10^4	10.	2.350	10^6	0.025	5.442
10^4	100.	2.352	10^6	0.05	6.180
10^5	0.025	3.045	10^6	0.1	6.914
10^5	0.05	3.428	10^6	0.2	7.599
10^5	0.1	3.811	10^6	1.	8.750
10^5	0.2	4.159	10^6	2.	8.967
10^5	1.	4.731	10^6	10.	9.087
10^5	10.	4.921	10^6	100.	9.095

One quantity of central importance to industrial applications is the total rate of heat transfer across the cavity. This is best represented by the Nusselt number defined at the sidewall as

$$\mathrm{Nu}=\int_0^1\left(\frac{\partial T}{\partial X}\right)_{x=0}\mathrm{d}Y$$

Figure 9 is representative of the qualitative nature of the evolution of Nu. The general trend of the time-dependent Nu, as portrayed in Figure 9, was also described by Ref. 2. However, Figure 9 clearly shows the effect of Pr. An oscillatory approach to steady state is seen for low values of Pr, whereas the Nu curve tends to the steady state in a smooth (and heavily damped) fashion for large values of Pr.

In order to cross-check the large volumes of numerical data with other investigations, the steady state values of Nu are listed in Table 1. These values are in close agreement with the available data from computational results of Markatos and Pericleous[18] and Mallinson and Davis.[19] Laborious efforts have been spent to delineate the effect of Pr on the steady-state value of Nu. At high Prandtl numbers, say $\mathrm{Pr}\gtrsim 10$, Nu is found to be quite insensitive to the variations of Pr. This implies that the analytical models formulated under the infinite Prandtl number can be effective for the realistic situations if $\mathrm{Pr}\gtrsim 10$. On the other hand, for low Prandtl numbers, the dependence of Nu on Pr is appreciable.

Conclusions

A comprehensive set of numerical solutions covering a wide range of parameter values for a square cavity has been obtained. When $\mathrm{Pr}\geq 1$, a distinct oscillatory behavior occurs if $\mathrm{Ra}>\mathrm{Pr}^4A^{-4}$. The period of oscillation is comparable to the period of an internal gravity wave. These findings are in support of the predictions of Patterson and Imberger. When $\mathrm{Pr}<1$, an oscillatory approach to the steady state is detected only when Ra is sufficiently high to render a strongly boundary-layer-type flow. The change in the horizontal surface conditions from insulated walls to highly conducting walls appears to have an insignificant influence on the time histories. The general pattern of the evolution of the Nusselt number based on the numerical solutions, is consistent with the results of Patterson and Imberger.

Acknowledgments

The authors are grateful to the anonymous referees. Their constructive comments led to substantial improvements of the paper. This work was supported in part by a research grant from the Korea Science and Engineering Foundation.

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